Lower Bound for Electron Spin Entanglement from Beam Splitter Current Correlations

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We determine a lower bound for the entanglement of formation of pairs of electron spins injected into a mesoscopic conductor. The bound can be expressed in terms of experimentally accessible quantities, the zero-frequency current correlators (shot noise power or cross correlators) after transmission through an electronic beam splitter and can be used to gain information about the entanglement from experiment. Spin relaxation (T_1 processes) and decoherence (T_2) during the ballistic coherent transmission of carriers are taken into account within Bloch theory. A variable inhomogeneous magnetic field gives rise to a useful lower bound for the entanglement of arbitrary states. The decrease in entanglement due to thermally mixed states is studied. Both the entanglement of the output of a source (entangler) and $T_{1,2}$ can be determined from current correlators.

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Quantum nonlocality has been an intriguing issue since the early days of quantum mechanics [1]. Nonlocal effects can come into play when a quantum system is composed of at least two subsystems (A and B) which are spatially separated. Despite their simplicity, the Bell states of two distant quantum two-state subsystems

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle), \tag{1}$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle), \tag{2}$$

exhibit the essential phenomenology of quantum nonlocality (e.g., they violate Bell's inequalities [2]) thus providing an ideal testing ground for quantum nonlocality. Here, we represent the two-state systems as spins 1/2 with basis states "spin up" $|\uparrow\rangle$ and "spin down" $|\downarrow\rangle$ with respect to an arbitrary fixed direction in space.

With the development of quantum information theory [3], and, in particular, with quantum communication, it has become clear that EPR pairs can also play the role of a *resource* for operations that are impossible with purely classical means. In this context, two-state systems are referred to as quantum bits (qubits), and quantum nonlocality is related to the concept of entanglement (defined below). A number of quantum information processes—quantum teleportation [4], quantum key distribution [5], quantum dense coding [6], etc.—have been successfully implemented using pairs of photons with entangled polarizations, i.e., in states such as Eqs. (1) and (2). Photons have the advantage of being easily moved from one place to another, allowing for experiments involving spacelike separations between detection events [2]. It is known that momentum-entangled photon pairs exhibit interference effects in the average intensity (particle number) after transmission through a beam splitter (BS) [7], an effect which can be used for Bell-state analysis [8].

More recently, there has been increasing interest in the use of electron spins in a solid-state environment for spin-based electronics [9] and as qubits for quantum computing [10]. Subsequently, quantum communication on a mesoscopic scale, typically micrometers in semiconductor structures, was proposed [11]. Rather than achieving spacelike separation between detection events on two sites (this would require subpicosecond detection), the idea here is to use quantum entanglement between parts of a coherently operating solid-state device (in the most extreme case, a quantum computer). It is then relevant to study the transport of spin-entangled electrons in a many-electron system and possible means of entanglement detection. Two-particle interference at a BS combined with the measurement of current fluctuations [12] (in general, the full counting statistics [13]), going beyond the average current, was identified as a detector for entanglement.

In this Letter, we go one step further, providing a *lower bound* for the *amount E* of spin entanglement carried by individual pairs of electrons, related to the zero-frequency current correlators when measured in a BS setup (Fig. 1, inset) by injecting the electrons separately into the two ingoing leads (1 and 2) and measuring either the autocorrelator $S_{\alpha\alpha}$ in one of the outgoing leads ($\alpha = 3, 4$) or the cross correlator S_{34}. Our result therefore relates experimentally accessible quantities with a measure for entanglement, the entanglement of formation E. Knowing E is important since it quantifies the usefulness of a bipartite state for quantum communication. We assume that the scattering region is smaller than both the coherence length and the mean free path, allowing for ballistic and coherent transport. In the following, T denotes the transmittivity of the BS, i.e., the probability to be scattered from lead 1 to lead 4 (or from 2 to 3). The ideal BS for the proposed setup does not give rise to

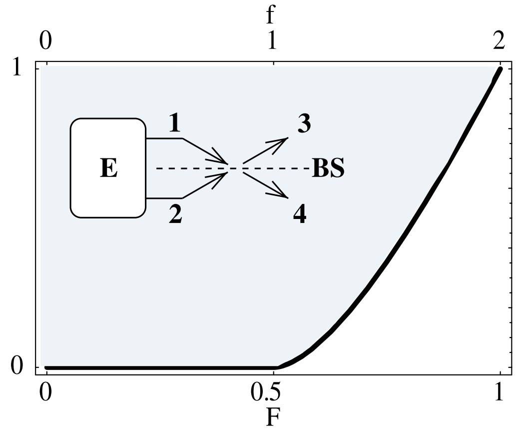

FIG. 1 (color online). Inset: Proposed setup with two-electron scattering at a beam splitter (BS) with transmittivity T. Electrons are injected pairwise from the entangler (E) into the BS contacts 1,2. The mean current $I = \langle I_\alpha \rangle$ and one of the correlators $S_{\alpha\beta}$ are measured at the contacts $\alpha, \beta = 3, 4$. Plot: Entanglement of formation E of the electron spins versus singlet fidelity F and the reduced correlator $f = S_{33}/2eIT(1-T)$. The curve illustrates the exact relation for Werner states. For general states, the curve is a lower bound for E; allowed values for E and f (or F) are above the curve.

backscattering (e.g., from lead 1 back into 1, or from 1 into 2, etc.). We also analyze the effect of such backscattering processes, as they give rise to background shot noise unrelated to entanglement. During transport, the spins are subject to decoherence and relaxation, e.g., caused by magnetic impurities, nuclear spins, or the spin-orbit coupling (see [14] for a review). We include these effects within a Bloch equation formalism [15]. Comparison between our theory and experiment will (i) test proposed entanglers [16–22] and (ii) determine spin relaxation (T_1) and decoherence (T_2) times. The materials and structures required for applying our theory, although at the forefront of current capabilities, appear to be feasible. The largest efforts seem to be necessary to realize the electron spin entangler [12] for which there exists a number of ideas, using normal [16,17] — or carbon-nanotube — superconductor junctions [18–20], or single [21], or coupled quantum dots [12,22]. The electronic BS and the measurement of BS current correlators have been demonstrated in a GaAs/AlGaAs heterostructure [23]. Coherent transport of spins over more than 100 μm in GaAs has been observed [24].

Traditionally, current correlations, and, in particular, shot noise, have been used to gain information about a scatterer beyond its conductance [25]. Here, we use a known scatterer (the BS) to gain information about the quantum state of the scattered particles. The correlation function between the currents $I_\alpha(t)$ and $I_\beta(t)$ in two leads $\alpha, \beta = 1, \ldots, 4$ of the BS is defined as

$$S^{\chi}_{\alpha\beta}(\omega) = \lim_{\tau\to\infty} \frac{h\nu}{\tau} \int_0^\tau dt\; e^{i\omega t}\, \mathrm{Re}\,\mathrm{Tr}[\delta I_\alpha(t) \delta I_\beta(0) \chi], \quad (3)$$

where $\delta I_\alpha = I_\alpha - \langle I_\alpha \rangle$, $\langle I_\alpha \rangle = \mathrm{Tr}(I_\alpha \chi)$, ν is the density of states in the leads, and χ is the density matrix of the injected electron pair (below, we suppress the orbital part of χ, being symmetric for $|\Psi_-\rangle$ and antisymmetric for all other Bell states; for Coulomb effects, see [12]). Writing χ in the singlet-triplet basis, $\chi = F|\Psi_-\rangle\langle\Psi_-| + G_0|\Psi_+\rangle\langle\Psi_+| + \sum_{i=\uparrow,\downarrow} G_i |ii\rangle\langle ii| + \Delta\chi$, where $\Delta\chi$ denotes off-diagonal terms, e.g., $|\Psi_-\rangle\langle\uparrow\uparrow|$, and $S_{\alpha\beta} \equiv S^{\chi}_{\alpha\beta}(\omega = 0)$, we arrive at $S_{\alpha\beta} = F S^{|\Psi_-\rangle}_{\alpha\beta} + G_0 S^{|\Psi_+\rangle}_{\alpha\beta} + \sum_{i=\uparrow,\downarrow} G_i S^{|ii\rangle}_{\alpha\beta}$, where $S^{|\Psi\rangle}_{\alpha\beta} \equiv S^{|\Psi\rangle\langle\Psi|}_{\alpha\beta}$. The off-diagonal terms in $\Delta\chi$ do not enter $S_{\alpha\beta}$ because the operators $\delta I_\alpha(t)$ conserve total spin. The coefficients F, G_0, $G_\uparrow$, and $G_\downarrow$ depend on the entangler [16–22]; it is the purpose of the proposed setup to gain information about them in order to determine the entanglement of χ. Using the standard scattering approach [25], we have found earlier [12] that the singlet state $|\Psi_-\rangle$ gives rise to enhanced shot noise (and cross correlators) at zero temperature, $S^{|\Psi_-\rangle}_{33} = -S^{|\Psi_-\rangle}_{34} = 2eIT(1-T)f$, with the *reduced correlator* $f = 2$, as compared to the "classical" Poissonian value $f = 1$ [26]. The average currents are given by $I = \langle I_3 \rangle = \langle I_4 \rangle = e/h\nu$. All triplet states are noiseless, $S^{|\Psi_+\rangle}_{\alpha\beta} = S^{|\uparrow\uparrow\rangle}_{\alpha\beta} = S^{|\downarrow\downarrow\rangle}_{\alpha\beta} = 0$ ($\alpha, \beta = 3, 4$). Both the auto correlations and cross correlations are only due to the singlet component of the incident two-particle state,

$$S_{33} = -S_{34} = F S^{|\Psi_-\rangle}_{33} = 2eIT(1-T)f, \qquad f = 2F. \quad (4)$$

Including backscattering with probability R_B, we find

$$S_{33} = 2eI[2F(1-R_B)T(1-T) + R_B/2], \quad (5)$$

$$S_{34} = -2eI\, 2F(1-R_B)T(1-T), \quad (6)$$

where $I = (e/h\nu)(1-R_B)$. Since $f' = S_{34}/2eIT(1-T) = 2F(1-R_B) \le f = 2F$ and $E(f)$ is monotonic (see below), we obtain a useful lower bound on E even for unknown R_B as long as $R_B < 1/2$. This does not hold for S_{33}. Alternatively, one can independently determine R_B by measuring the noise using Fermi leads [23].

The entanglement of a bipartite *pure* state $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ is given by the von Neumann entropy $S_N(|\psi\rangle) = -\mathrm{Tr}_B \rho_B \log \rho_B$ (log in base 2) of $\rho_B = \mathrm{Tr}_A |\psi\rangle\langle\psi|$, where $0 \le S_N \le 1$, $S_N(|\Psi_\pm\rangle) = S_N(|\Phi_\pm\rangle) = 1$, and $S_N(|\psi\rangle) = 0 \Leftrightarrow |\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$. Physically, if $S_N(|\psi\rangle) \simeq N/M$ then $M \ge N$ copies of $|\psi\rangle$ are sufficient to perform, e.g., quantum teleportation of N qubits for $N, M \gg 1$ (similarly for other quantum communication protocols). Generally, for a bipartite *mixed* state χ the entanglement of formation [27] is $E(\chi) = \min_{\{(|\chi_i\rangle, p_i)\} \in \mathcal{E}(\chi)} \sum_i p_i S_N(|\chi_i\rangle)$, where $\mathcal{E}(\chi) = \{(|\chi_i\rangle, p_i) | \sum_i p_i |\chi_i\rangle\langle\chi_i| = \chi\}$, i.e., the least expected entanglement of any ensemble of pure states realizing χ. A state with $E > 0$ ($E = 1$) is (maximally) entangled, and neither local operations nor classical communication (LOCC) between A and B can increase E.

For arbitrary χ, $E(\chi)$ is not a function of only the singlet fidelity $F = \langle\Psi_-|\chi|\Psi_-\rangle$. However, $E(\chi) = E(F)$ for the Werner states [28]

$$\rho_F = F|\Psi_-\rangle\langle\Psi_-| + \frac{1-F}{3}\Big(|\Psi_+\rangle\langle\Psi_+| + \sum_{i=\pm}|\Phi_i\rangle\langle\Phi_i|\Big), \quad (7)$$

the unique rotationally invariant states with singlet fidelity F. It is known [27] that $E(F) \equiv E(\rho_F) = H_2[1/2 + \sqrt{F(1-F)}]$ if $1/2 < F \leq 1$ and $E(F) \equiv E(\rho_F) = 0$ if $0 \leq F < 1/2$, with $H_2(x) = -x\log x - (1-x)\log(1-x)$. With Eq. (4), we can express $E(\rho_F)$ in terms of the reduced correlator f (Fig. 1).

We generalize this result to arbitrary mixed states χ of two spins (qubits). Any state χ can be transformed into ρ_F with $F = \langle\Psi_-|\chi|\Psi_-\rangle$ by a random bipartite rotation [27,29], i.e., by applying $U \otimes U$ with a random $U \in$ SU(2). Since this operation involves only LOCC,

$$E(F) \leq E(\chi). \quad (8)$$

The entanglement of formation $E(F)$ of the corresponding Werner state therefore provides a *lower bound* on $E(\chi)$ (Fig. 1). Thus, a noise signal $f = 2F > 1$ in the BS setup can be interpreted as a sign of entanglement, $E(F) > 0$, between the spins injected into leads 1 and 2 [30].

We now include relaxation and decoherence into our analysis. At time $t = 0$, we start with a spin singlet (upper sign) or triplet (lower sign) state

$$\chi(0) = |\Psi_\mp\rangle\langle\Psi_\mp|. \quad (9)$$

We describe the dynamics of $\chi(t)$ in a field $\mathbf{B} \parallel \hat{z}$ and in the presence of decoherence and relaxation within a single-spin Bloch equation for the polarization $\mathbf{P} = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$,

$$\dot{\mathbf{P}} = \mathbf{P} \times \mathbf{h} - R(\mathbf{P} - \tilde{\mathbf{P}}) \equiv -\Omega(\mathbf{P} - \tilde{\mathbf{P}}), \quad (10)$$

with $\langle\sigma_i\rangle = \mathrm{Tr}(\sigma_i\rho)$, $\mathbf{h} = g\mu_B\mathbf{B} = (0, 0, h)$, $\tilde{\mathbf{P}} = (0, 0, \tilde{P})$ (note that $\tilde{\mathbf{P}} \times \mathbf{h} = 0$), and the relaxation matrix [31] $R_{ij} = \delta_{ij}R_i$ with $R_1 = R_2 = T_2^{-1}$ and $R_3 = T_1^{-1}$. Solving Eq. (10), we obtain $\mathbf{P}(t) = e^{-\Omega t}\mathbf{P}(0) + (1 - e^{-\Omega t})\tilde{\mathbf{P}}$, or, in terms of the single-spin density matrix,

$$\rho(t) = [P_0 + \mathbf{P}(t) \cdot \boldsymbol{\sigma}]/2 \equiv \Lambda_h(t)[\rho(0)], \quad (11)$$

with the superoperator $[a(t) = 1 - e^{-t/T_1}]$ [32]

$$\Lambda_h(t)[\rho] = \begin{bmatrix} \frac{1}{2}(\rho_{\uparrow\uparrow} + \rho_{\downarrow\downarrow})[1 + a(t)\tilde{P}] + \frac{1}{2}(\rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow})e^{-t/T_1} & e^{-t/T_2 + iht}\rho_{\uparrow\downarrow} \\ e^{-t/T_2 - iht}\rho_{\downarrow\uparrow} & \frac{1}{2}(\rho_{\uparrow\uparrow} + \rho_{\downarrow\downarrow})[1 - a(t)\tilde{P}] - \frac{1}{2}(\rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow})e^{-t/T_1} \end{bmatrix}, \quad (12)$$

with $\rho_{ij} = \langle i|\rho|j\rangle$ and $(i, j = \uparrow, \downarrow)$. We apply $\Lambda_h(t_0)$ to both spins individually,

$$\chi(t_0) = [\Lambda_{h_1}(t_0) \otimes \Lambda_{h_2}(t_0)][\chi(0)], \quad (13)$$

where h_i is the field at electron i and $t_0 = L/v_F$ is the *fixed* ballistic transmission time (L = length of ingoing leads, v_F = Fermi velocity). We are describing a stationary state in the BS ($\omega = 0$), and there is no time dependence in $\chi(t_0)$, t_0 being fixed. For a typical GaAs structure with $L \approx 1\ \mu$m and $v_F \approx 10^4$–10^5 m/s, we obtain $t_0 \approx 10$–100 ps [26]. With typical decoherence times up to $T_2 \approx 100$ ns–1 μs found in GaAs we obtain ratios T_2/t_0 up to 10^3–10^5 (typically, $T_1 \gg T_2$). Using Eq. (4) and $F(t_0) = \langle\Psi_-|\chi(t_0)|\Psi_-\rangle$ we obtain

$$f(t_0) = \pm e^{-2t_0/T_2}\cos(\delta h\, t_0) + \frac{1}{2}(1 + e^{-2t_0/T_1}) - \frac{1}{2}(1 - e^{-t_0/T_1})^2\tilde{P}^2, \quad (14)$$

where $\delta h = h_1 - h_2$. If the decoherence times $T_2^{(1,2)}$ of the two electrons are different, then $T_2/2$ in Eq. (14) becomes $T_2^{\mathrm{EPR}} = [1/T_2^{(1)} + 1/T_2^{(2)}]^{-1}$. We define T_1^{EPR} similarly if $\tilde{P} = 0$. However, if $\tilde{P} = 1$, then $\exp(-t_0/T_1)$ is replaced by $\exp[-t_0/T_1^{(1)}] + \exp[-t_0/T_1^{(2)}]$.

A homogeneous field, $\delta h = 0$, does not affect f. For slow relaxation, $T_1 \gg t_0$, we find $f(t_0) = 1 \pm e^{-2t_0/T_2}$ [see Fig. 2(a)]. For unentangled triplet states, $\chi(0) = |\uparrow\uparrow\rangle\langle\uparrow\uparrow|, |\downarrow\downarrow\rangle\langle\downarrow\downarrow|$, we find $f \leq 1/2$ for all T_1, T_2, and $\tilde{P}$ [Fig. 2(b)]. An inhomogeneous field $\delta h \neq 0$ (or, equivalently, a local controllable Rashba spin-orbit coupling [33]) has the effect of continuously rotating singlets into triplets and vice versa (Fig. 3). This provides a lower bound of the triplet entanglement, $\chi(0) = |\Psi_+\rangle\langle\Psi_+|$, which is as tight as Eq. (8) for the singlet,

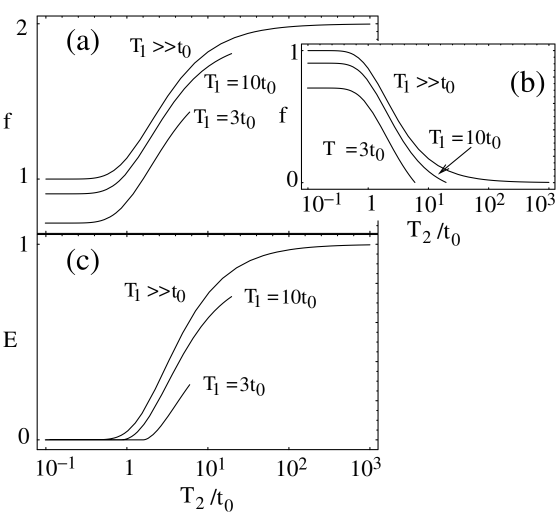

FIG. 2. Homogeneous field $\delta h = 0$ and $\tilde{P} = 1$. (a) f of the singlet state $|\Psi_-\rangle$ after ballistic transmission through a BS as a function of T_2/t_0, where $t_0 = L/v_F$. Different curves correspond to different values of the relaxation time T_1/t_0 (plotted only for $T_2 \leq 2T_1$). (b) f for a triplet state $|\Psi_+\rangle$. Since $f \leq 1$, the lower bound on E is zero, i.e., determination of triplet entanglement is impossible at $\delta h = 0$. (c) Lower bound on E.

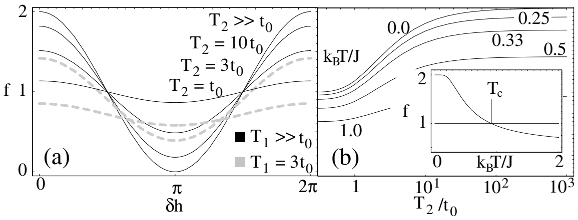

FIG. 3. (a) Reduced current correlator f versus field inhomogeneity $\delta h = h_1 - h_2$ (units of $\hbar/t_0 g\mu_B$, $t_0 = L/v_F$, $g =$ g factor, $\mu_B =$ Bohr magneton) for injected singlet and $\tilde{P} = 1$. Solid lines represent $T_1 \gg t_0$ and $T_2 = t_0, 3t_0, 10t_0, \infty$, gray dashed lines $T_1 = 3t_0$ and $T_2 = t_0, 3t_0$. For triplets, the plot is phase shifted by π, providing tight lower bounds at $\delta h = \pi$. Tight bounds for *any* input state are obtained by varying the direction of $\delta\mathbf{h}$. (b) Plot of f for a thermally mixed initial state versus T_2/t_0 for $T_1 \gg t_0$, $\tilde{P} = 1$. Various curves correspond to $k_BT/J = 0, 0.25, 0.33, 0.5, 1$, where J and T denote the exchange energy and temperature during state preparation. Inset: The maximal f (at $T_1, T_2 \gg t_0$) versus k_BT/J. Entanglement is absent ($f \leq 1$, $E = 0$) above $T_c = 0.91\,J/k_B$.

$$E(\chi) \geq \max_{\delta\mathbf{h}} E[f(\delta\mathbf{h})/2], \qquad (15)$$

where $f(\delta\mathbf{h})$ is the measured noise and $E(F)$ is the entanglement of the Werner state ρ_F (Fig. 1). If a field inhomogeneity $\delta\mathbf{h}$ can be created pointing in *arbitrary* directions in space, then Eq. (15) is a tight lower bound for *any* injected entangled state. In particular, each maximally entangled state $|\Psi\rangle$ will be detected in this way, since there exists a $u = \exp(-i\delta\mathbf{h}\cdot\boldsymbol{\sigma}) \in \mathrm{SU}(2)$ such that $u^\dagger \otimes u|\Psi\rangle = |\Psi_-\rangle$. This rotation can also be done unilaterally [33], i.e., there is a $v \in \mathrm{SU}(2)$ with $v \otimes 1|\Psi\rangle = |\Psi_-\rangle$. The field gradient $\delta\mathbf{h}$ required to perform a π rotation in GaAs ($g = -0.44$) is of the order of 1 T, but the same rotation can be obtained from the Rashba effect on a length scale of about 70 nm [26].

Finally, we study the case where $\chi(0)$ is mixed, being prepared at a temperature T comparable to the energy splitting between spin states, typically (if the Zeeman effect is negligible) the exchange energy J, i.e., the singlet-triplet splitting. In this case, $\chi(0) = \rho_F$ with $F = (1 + 3e^{-J/k_BT})^{-1}$ where k_B is Boltzmann's constant. We show only the resulting f for $T_1 \gg t_0$ here (the full expression will be reported elsewhere [34]), $f(t_0) = [1 + e^{-2t_0/T_2} + e^{-J/k_BT}(1 - e^{-2t_0/T_2})]/(1 + 3e^{-J/k_BT})$, which is the statistical mixture of Eq. (14) for the singlet and triplet with the appropriate Boltzmann weights [Fig. 3(b)]. Above the critical temperature $T_c = 0.91\,J/k_B$ there is no entanglement even for $T_1, T_2 \to \infty$.

Note that intensity correlators at an optical BS (half-silvered mirror) determine a lower bound for the entanglement of formation between the polarization of photons in the same way as for electrons in a conductor.

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