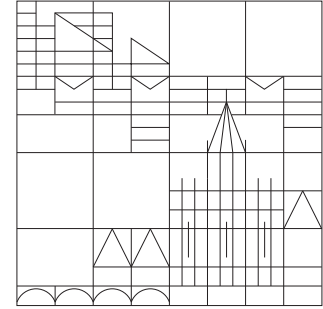


UNIVERSITÄT KONSTANZ  
 Fachbereich Physik  
 Prof. Dr. Guido Burkard  
 Dr. Regina Finsterhoelzl  
 Dr. Balázs Gulácsi  
<https://tinyurl.com/QC-WS23>



**Quantencomputing und Quantensimulation**  
**Wintersemester 2023 - Übungsblatt 11**

Ausgabe: 26.01.2024, Abgabe: 02.02.2024, Übungen: 09.02.2024

**Bonus-Aufgabe 26: Diffie-Hellman key exchange (3 bonuspoints)**

The Diffie-Hellman key exchange describes a method by which two people (Alice and Bob) can exchange a common key via an insecure (interceptable) channel, which can then be used to encrypt further messages. First, Alice and Bob agree on a large prime number  $p$  and another small number  $g$ , which are exchanged via the insecure channel. Now both Alice and Bob choose a random secret number  $a$  and  $b$  with  $0 \leq a, b \leq p - 1$ , which they each keep to themselves. Alice now calculates  $A = g^a \text{ mod } p$  and sends the result to Bob. Bob calculates  $B = g^b \text{ mod } p$  and sends the result to Alice. Alice now calculates the key  $K$  by  $K = B^a \text{ mod } p$ . Bob receives the same key by  $K = A^b \text{ mod } p$ .

- a) (1 point) Calculate an example for  $g = 21$  and  $p = 101$ .
- b) (1 point) Show that  $B^a \text{ mod } p = A^b \text{ mod } p$ .
- c) (1 point) The Diffie-Hellman key exchange is considered secure for sufficiently large prime numbers  $p$ . Describe how Eve could still calculate the secret key using a quantum computer by listening to the channel.

**Bonus-Aufgabe 27: Implementing Grover’s algorithm (3 bonuspoints)**

Consider the circuit shown to implement an oracle  $U_f$  for 3 qubits. Sketch a circuit to find the desired states. How many iterations of the Grover algorithm are required? Which states are marked by the oracle?

*Note: 5 extra points for implementing this problem on a real quantum computer, for example [IBM Quantum](#) or [Quantum Inspire](#).*

