Weak-measurement theory of quantum-dot spin qubits

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(Received 1 June 2007; published 25 October 2007)

The theory of weak quantum measurements is developed for quantum-dot spin qubits. Building on recent experiments, we propose a control cycle to prepare, manipulate, weakly measure, and perform quantum state tomography. This is accomplished using a combination of the physics of electron spin resonance, spin blockade, and Coulomb blockade, resulting in a charge transport process. We investigate the influence of the surrounding nuclear spin environment, and find a regime where this environment significantly simplifies the dynamics of the weak-measurement process, making this theoretical proposal realistic with existing experimental technology. We further consider spin-echo refocusing to combat dephasing, as well as discuss a realization of "quantum undemolition," whereby the effects of quantum state disturbance are undone.

DOI: 10.1103/PhysRevB.76.155324 PACS number(s): 03.65.Ta, 03.67.Lx, 73.63.Kv, 76.30.−v

I. INTRODUCTION

Continuous weak measurement has attracted great interest recently, not only because the phenomenon sheds light on fundamental physics, but also for its possible application to practical tasks in computation, state preparation, and error correction. While the informational theory of weak measurement has been under active development in quantum-dot charge qubits,[1–3] the theory has not been well developed for spin qubits, a major area of experimental activity. The purpose of this paper is to develop the theory of weak measurements for spin qubits, both regarding the manner in which the state is affected by weak measurement and applications that can be developed with controlled weak quantum measurements. Importantly, the modern theory of weak measurements has recently been experimentally verified in the solid state by the Martinis group. This experiment investigated weak measurements in superconducting phase qubits by utilizing quantum state tomography of the postmeasurement state.[4]

The use of electron spins as quantum bits is very attractive in view of their ability to be effectively isolated from the environment for relatively long times.[5] These long coherence times are due in part to the small magnetic moment of the electron. A small magnetic moment also poses a problem for single spin readout. This was overcome by the use of spin-to-charge conversion,[6,7] a technique[5] where the spin information is first converted into charge information which is subsequently detected, using, e.g., a quantum point contact. A second major problem is how to couple two nearby spins, considering the very weak direct magnetic dipole interaction. This difficulty was overcome by using the charge-mediated exchange coupling.[8] The latest experimental accomplishment demonstrates single-spin manipulation with (magnetic) electron spin resonance (ESR).[9] In this experiment, it was shown that short bursts of oscillating magnetic field can drive coherent Rabi oscillations in the individual electron spins confined to a quantum dot.

All the ingredients for universal quantum computation are now available in this system. However, there has been recent theoretical activity indicating that there may be significant practical advantages to using weak continuous measurement over projective measurements. For example, it has been shown that rather than using two-qubit unitary operations plus projective single-qubit measurements, a two-qubit parity meter[10–13] (where only the parity subspace of the two-qubit Hilbert space is able to be resolved) plus single-qubit measurements is sufficient to enable universal quantum computation[10–13] as well as create fully entangled Bell states.[12–15] This discovery eliminates the need for two-qubit unitaries thus avoiding the necessity of strong (direct) qubit-qubit interactions. This one example is sufficient impetus to justify intensive investigation into weak measurements for spin qubits.

We will now describe some of the details of the recent experiment of Koppens *et al.* that we propose to extend.[9] The qubit is encoded with two electron spins, where each electron is confined in a separate quantum dot. Electrical bias is applied across the double quantum dot, where the right dot is lowered energetically below the left dot with a gate voltage. The notation (n,m) refers to n electrons occupying the left dot, and m electrons occupying the right dot. Electrons can tunnel from the left lead to the left dot with a rate Γ_L. The dots are tuned to the Coulomb-blockade (CB) regime such that only the states (0, 1), (1, 1), and (0, 2) can be occupied during a transport cycle. The gate voltages applied to the quantum dot structure are tuned such that the sequential tunneling cycle $(0,1)\rightarrow(1,1)\rightarrow(0,2)\rightarrow(0,1)$ is energetically allowed. This cycle consists of a first step, where an electron hops onto the left quantum dot, a second step where an electron hops from the left to the right dot (which has been occupied previously by a single electron) with rate Γ, and finally a third step which closes the cycle and in which one of the electrons on the right dot hops out into the right reservoir with rate Γ_R. In this sequential tunneling configuration, *spin blockade* further restricts transport to situations where the two electrons form a spin singlet $(0,2)S$ on the right dot while the spin triplet $(0,2)T$ is outside the transport energy window due to the large single-quantum-dot exchange energy J, see Fig. 1(B). If the electrons are in any of

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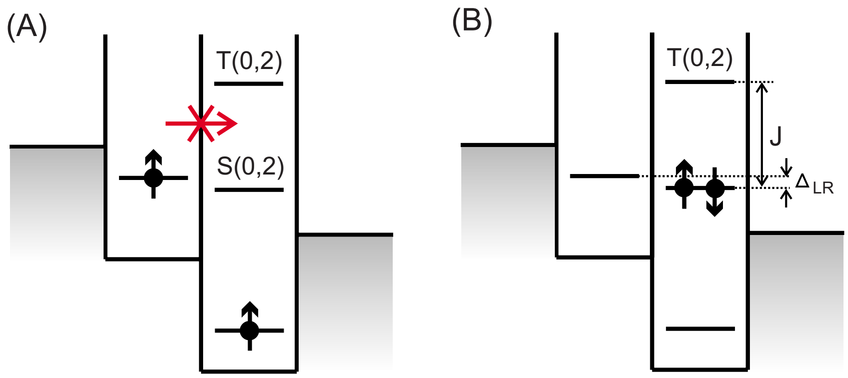

FIG. 1. (Color online) The figure illustrates the two possible two-electron double dot states that appear in the weak-measurement setup (as described in the text). (A) The double dot is in the (1,1) configuration and the two electron spins form a triplet. Then, the electron in the left dot cannot tunnel into the right dot to form a (0,2) state because the triplet state $T(0,2)$ of the (0,2) configuration is energetically too high, putting it outside the transport energy window. The energy difference between $T(0,2)$ and $S(0,2)$ is the spin exchange coupling J. The energy difference Δ_{LR} between the (1,1) and the $S(0,2)$ states (which is chosen to be much smaller than eV in the figure, where V is the applied bias) can be tuned by external gates that shift the energy levels of each dot independently. Therefore, Δ_{LR} can, in principle, take any desired value. (B) If the two electron spins in the (1,1) configuration form a singlet $S(0,2)$, then tunneling from the (1,1) to the (0,2) configuration is energetically allowed and the resulting state is shown. It is possible to unblock the state shown in (A) and to allow for the transition to the state shown in (B) by applying an electron spin resonance (ESR) signal to the left dot while the two electrons are in the spin-blocked state $T(1,1)$.

the triplet states $(1,1)T$, then the current is blocked since the electron in the left dot can neither tunnel to the right nor to the left as illustrated in Fig. 1(A). Once this spin-blockade state is reached, the gate voltages are adjusted such that the system is now in the CB regime, where sequential transport is suppressed by the interaction between the electrons and the occupation numbers on the dots are fixed to (1,1). Then, an ESR pulse is used to prepare a superposition of the singlet and triplet states (Fig. 2). In the CB regime, any unwanted tunneling events between the dots and the leads that could lead to spin flips are suppressed. After the ESR pulse, the system is brought back into the sequential transport regime which now allows for a coherent weak measurement of the prepared state.

This setup is naturally suited to investigate weak quantum measurements. The measurement scheme we now describe is closely related to recent developments in superconducting phase qubits[4,16] where the readout process also involves a quantum tunneling process. The essential idea is to introduce another time scale into the measurement process. By waiting for a time much longer than the average interdot tunneling time Γ^{-1}, one projects the system with certainty into either the triplet subspace, or the singlet state. However, if it is possible to let the system "try to tunnel" for a time comparable to Γ^{-1}, then the measurement is weak. We will give the details of how this happens below.

The physical process described above may be mathematically described by introducing a measurement operator M_Q that describes the physical weak measurement experienced by the spin qubit, such that the probability of either event given an initial density matrix ρ is given by

$$P(Q) = \mathrm{Tr}\, \rho M_Q^\dagger M_Q, \tag{1}$$

where $Q=0$ if no electron has tunneled and $Q=1$ if an electron has tunneled. Quantum mechanics then predicts that coherent, yet nonunitary evolution of the density matrix under the condition that measurement result Q is found is given by

$$\rho' = M_Q \rho M_Q^\dagger / P(Q), \tag{2}$$

(see, e.g., Ref. 18) where the positive operator-valued measure (POVM) elements $E_Q = M_Q^\dagger M_Q$ must obey completeness, $\Sigma_Q E_Q = 1$. One of the main differences compared to the superconducting phase qubit example already demonstrated[4] is

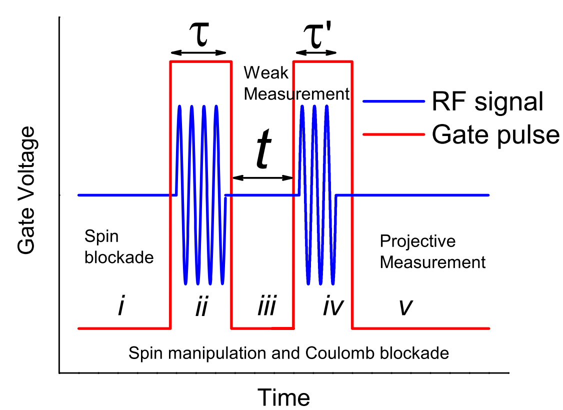

FIG. 2. (Color online) Control cycle for the weak-measurement demonstration. (i) The system is brought into the spin-blockade regime, see Fig. 1(A). (ii) A gate pulse is used to tune the system into Coulomb blockade and a rf signal is applied. This is the state preparation step. (iii) In the weak-measurement step, tunneling from the left to the right dot is possible if the system is in the singlet configuration. This step lasts shorter than Γ^{-1}. (iv) A gate pulse is used to tune the system into Coulomb blockade and a rf signal is applied. This is the state tomography step. (v) In the projective measurement step, tunneling from the left to the right dot is possible if the system was in the singlet configuration. This step lasts longer than Γ^{-1}.

the fact that the informational spin qubit is encoded into two microscopic electrons (two physical qubits), rather than just one macroscopic qubit. Another difference is the fact that the spin readout mechanism is via a charge transport process, rather than a change of magnetic flux.

We anticipate that the development of the theory of weak quantum measurements for spin qubits will play a key role in future experimental investigations, as full quantum control is mastered.

II. MINIMAL MODEL

We first consider the simplest case of no environmental decoherence from the surrounding nuclear spins, and no inelastic transitions. We also assume for simplicity that $\Gamma \ll \Gamma_L, \Gamma_R$, so the central barrier is the bottleneck in the transport cycle. When both dots are occupied by one electron, (1,1), we define the triplet (T) and singlet (S) states as

$$T_0 = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, \quad T_+ = |\uparrow\uparrow\rangle, \quad T_- = |\downarrow\downarrow\rangle,$$

$$S = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}. \tag{3}$$

We now follow a modification of the control cycle described in Ref. 9 (see Fig. 2) as follows.

(i) We consider the initial state to be T_+ (which of the triplet states is chosen is not really important), so the transport cycle is blocked.

(ii) Next, we lower the gate voltage on the left dot, putting the system into Coulomb blockade (none of the levels in the right dot are accessible, forbidding all transitions), and turn on the ESR signal that induces single spin rotations on the left qubit (here we assume the ESR pulse is on resonance with the left spin only, and make the rotating wave approximation). In particular, in the rotating frame

$$|\uparrow\rangle_L \rightarrow \cos\theta_1 |\uparrow\rangle_L + \sin\theta_1 |\downarrow\rangle_L, \tag{4}$$

where $\theta_1 = \Omega\tau$, Ω is the Rabi flopping frequency, and τ is the time it is on for. Therefore, after the ESR pulse, the two-spin state is $|\psi_0\rangle = \cos\theta_1 |\uparrow\uparrow\rangle + \sin\theta_1 |\downarrow\uparrow\rangle = \cos\theta_1 T_+ + \sin\theta_1 (T_0 - S)/\sqrt{2}$.

(iii) The next step is the raising of the left gate voltage, allowing the system to tunnel in a state-selective way. After the left electron enters the right dot [forming the state $S(0,2)$], the much larger right tunneling rate causes the escape of the electron to the right lead, leaving (0,1). As mentioned before, the transitions $T_0(1,1) \rightarrow S(0,2), T(0,2)$ are forbidden because of either conservation of spin [for $S(0,2)$] or conservation of energy [for $T(0,2)$]. These forbidden processes give no transported charge, $Q=0$, but the transition $S(1,1) \rightarrow S(0,2)$ is allowed (giving a transported charge, $Q=1$) with rate Γ. Because tunneling is an exponential decay process, we can express this mathematically by saying that in the singlet-triplet (ST) basis (T_+, T_-, T_0, S), the singlet-singlet matrix element of the POVM element is

$$\langle S|M_Q^\dagger M_Q|S\rangle = \begin{cases} \exp(-\Gamma t), & Q=0 \\ 1-\exp(-\Gamma t), & Q=1, \end{cases} \tag{5}$$

while the triplet-triplet matrix elements are

$$\langle T_j|M_Q^\dagger M_Q|T_j\rangle = \begin{cases} 1, & Q=0 \\ 0, & Q=1, \end{cases} \tag{6}$$

where $j=+,-,0$. The off-diagonal matrix elements vanish in this basis. Considering a pure tunneling process that does not induce any phase, we can write the measurement operators in the ST basis simply as the square root of the POVM element.

Therefore, the postmeasurement state of the qubit is (for pure states) $\psi'_Q = M_Q \psi_i / \mathcal{N}$, where $\mathcal{N}$ is the (re-)normalization of the new state, so that $P_{Q=0} = |\mathcal{N}|^2$. If $Q=1$, the state is destroyed [the configuration is now (0,2) which will quickly be followed by the electron tunneling and going to the drain]. The probability of finding an electron in the drain ($Q=1$) at this step is then

$$P_{iii}(1) = \langle\psi_0|M_1^\dagger M_1|\psi_0\rangle = \sin^2\theta_1 [1-\exp(-\Gamma t)]/2. \tag{7}$$

The probability of not finding an electron in the drain ($Q=0$) at this step is

$$P_{iii}(0) = \langle\psi_0|M_0^\dagger M_0|\psi_0\rangle = \cos^2\theta_1 + \sin^2\theta_1 [1+\exp(-\Gamma t)]/2. \tag{8}$$

Notice that $P_{iii}(0) + P_{iii}(1) = \cos^2\theta_1 + \sin^2\theta_1 = 1$. In the null-result ($Q=0$) case, the postmeasurement state is

$$\psi'_0 = \frac{1}{\sqrt{2}\mathcal{N}} \begin{pmatrix} \sqrt{2}\cos\theta_1 \\ 0 \\ \sin\theta_1 \\ -\sin\theta_1 \exp(-\Gamma t/2) \end{pmatrix}, \tag{9}$$

where $\mathcal{N}^2 = \cos^2\theta_1 + \sin^2\theta_1 D_+$, and we define

$$D_\pm = [1 \pm \exp(-\Gamma t/2)]/2. \tag{10}$$

If no time has elapsed, then the new state is identical to the initial state, while in the long time limit, $\Gamma t \gg 1$, the singlet portion of the state is continuously removed. Qualitatively, this is because if no charge is seen to be transferred after a sufficiently long time, we can be confident that the quantum state must be somewhere in the triplet subspace, but we gain no information about which triplet state the system is in.

(iv) In order to confirm that the state (9) is indeed the postmeasurement state, we can apply quantum state tomography by first applying another ESR pulse, and then a second (projective) measurement.[17] Because the ESR pulse acts in the left-right (LR) basis, and not the ST basis, it is first necessary to return to the LR basis. The unitary operation U_2 that converts the basis (T_+, T_-, T_0, S) to $(|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)$ is

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

Applying this matrix to the state (9), we find that in the LR basis

$$\psi'_{L/R} = \frac{1}{\mathcal{N}} \begin{pmatrix} \cos\theta_1 \\ 0 \\ \sin\theta_1 D_- \\ \sin\theta_1 D_+ \end{pmatrix}. \quad (12)$$

We can now implement the ESR pulse on the left spin by applying the SU(2) rotation matrix

$$R_L = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \quad (13)$$

to the left spin only, where $\theta_2 = \Omega\tau'$ represents the angle the left spin is driven through in the rotating frame. This produces the state

$$\psi^{\text{final}}_{L/R} = \frac{1}{\mathcal{N}} \begin{pmatrix} \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 D_+ \\ \sin\theta_1 \sin\theta_2 D_- \\ \sin\theta_1 \cos\theta_2 D_- \\ \cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2 D_+ \end{pmatrix}. \quad (14)$$

(v) Make a projective measurement: Now we lower the gate voltage again, allowing the left electron to tunnel to the right well (and escape to the drain), this time keeping the voltage low for a time much longer than the inverse tunneling rate. The probability that the tunneling event will occur is given by the square overlap between the state (14) and the singlet state, $P_v(1) = |\langle S|\psi^{\text{final}}_{L/R}\rangle|^2$. The final state in the ST basis is given by applying the inverse of U_2, so we find the following for the probability $P_v(1)$ of tunneling in the second (strong) measurement:

$$P_v(1) = \frac{[\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2 \exp(-\Gamma t/2)]^2}{2P_{iii}(0)}, \quad (15)$$

where we recall $P_{iii}(0) = \mathcal{N}^2 = \cos^2\theta_1 + \sin^2\theta_1 D_+$.

We are now in a position to compute the total probability of finding a transported electron through the whole cycle. This is given by the probability the tunneling event occurred in step (iii) or the probability the tunneling event did not occur in step (iii), but did occur in step (v). Therefore the total probability is given by

$$\begin{aligned} P_{\text{tot}} &= P_{iii}(1) + P_{iii}(0)P_v(1) \\ &= \sin^2\theta_1[1 - \exp(-\Gamma t)]/2 + [\cos\theta_1 \sin\theta_2 \\ &\quad + \sin\theta_1 \cos\theta_2 \exp(-\Gamma t/2)]^2/2. \end{aligned} \quad (16)$$

This result is naturally interpreted in terms of a state preparation step, characterized by θ_1, the weak measurement, characterized by a strength Γt, and a tomography step, characterized by an angle θ_2.

This analysis describes one cycle. The experiment is now repeated many times, with a cycle period T, and the average current is measured at fixed weak-measurement times, and rotation angles. The average current is given by the total probability of a successful tunneling event, divided by the cycle time,

$$\langle I \rangle = \frac{eP_{\text{tot}}}{T}. \quad (17)$$

Here we see another attractive feature of the proposal: there is no need for statistical averaging over large data sets as in Ref. 4; the system self-averages and gives the final answer [Eqs. (16) and (17)] as a small electrical current.

Generalizing to the situation where the initial state is in any coherent superposition $\psi_0 = \alpha T_0 + \beta T_+ + \gamma T_-$ of triplet states and repeating the previous steps, we find the total probability is given by that same result, Eq. (16), but weighted by the overall factor $|\beta + \gamma|^2$. This indicates that if the initial state was T_0, no electrons could be transferred with this sequence.

There is the possibility that between cycles there can be the uninterrupted cycle: $(0,1) \to S(1,1) \to S(0,2) \to (0,1)$. This will contribute a background current to the signal that must be subtracted. We note that the choice $\theta_1 = \theta_2 = \pi$ gives a vanishing signal for all weak-measurement times t, providing a calibration point.

In the transport cycle, the system will get blocked in statistically independent triplet states, and therefore we should average the total tunneling probability over an ensemble of initial triplet states. Taking the average with a completely mixed density matrix indicates that $\langle|\alpha|^2\rangle_r = \langle|\beta|^2\rangle_r = \langle|\gamma|^2\rangle_r = 1/3$, where $\langle\cdots\rangle_r$ denotes averaging over statistically independent realizations, while the coherences average to zero in repeated realizations. Applying this average to the general probability, we find again the result (16), but multiplied by $2/3$.

III. INFLUENCE OF THE INHOMOGENEOUS NUCLEAR MAGNETIC FIELD

We now turn to a more realistic treatment of the physics by including the effect of the surrounding environment. The dominant source of dephasing in GaAs quantum-dot spin qubit is interaction with the surrounding nuclear spins. This has been theoretically analyzed in detail in the past few years[19–22] and also experimentally verified.[23,24] The dynamics of the nuclear system is much slower than the electron spin dynamics, so in a given run the composite nuclear magnetic field is essentially static. This gives rise to a systematic (unknown) unitary rotation which is taken into account below. The magnetic field changes in different realizations of the measurement cycle, leading to an effective dephasing when the data are averaged over a statistical ensemble. We will discuss dephasing in more detail below.

To take these effects into account, the Hamiltonian of the spins interacting with the external $\mathbf{B}_{\text{ext}}$, nuclear, and oscillating $\mathbf{B}_{\text{ac}}$ magnetic field is

$$H = g\mu_B(\mathbf{B}_{\text{ext}} + \mathbf{B}_{L,N})\mathbf{S}_L + g\mu_B(\mathbf{B}_{\text{ext}} + \mathbf{B}_{R,N})\mathbf{S}_R + g\mu_B \cos(\Omega t)\mathbf{B}_{\text{ac}}(g_L\mathbf{S}_L + g_R\mathbf{S}_R). \quad (18)$$

Here $\mathbf{B}_{L,N}$ and $\mathbf{B}_{R,N}$ are the nuclear fields in left and right dots that are static in a given run, $\mathbf{S}_L$ and $\mathbf{S}_R$ are the left and right spin operators. Orienting the external field in the z direction, the x and y components of the nuclear field tend to admix T_-, T_+, and S. Because of the large energy difference between these states in the presence of the large external magnetic field $|\mathbf{B}_{\text{ext}}| \gg |\mathbf{B}_{L,N}|, |\mathbf{B}_{R,N}|$, these transitions are suppressed. However, the z component of the nuclear field causes S and T_0 to admix (out of the rotating frame) with a time scale $\tau_{\text{admix}} = 1/(B_N g \mu_B)$. This time scale can in practice be larger or smaller than the inverse tunneling rate, Γ^{-1}.

The magnetic field from the nuclear spins also causes the ESR pulse to be usually on resonance with only one of the spins. We will now consider the two limiting cases, where tunneling is much faster or much slower than the S, T_0 admixing time.

A. Slow triplet-singlet admixing

In the first case where $T \gg \tau_{\text{admix}} \gg \Gamma^{-1}$, the analysis of the previous section is applicable, with the exception that the time T between successive cycles is much longer than τ_{admix}, so the T_0 component of the state will have time to admix with S and subsequently tunnel out (as was the case in the experiment).[9] In this regime, we have the case where the weak measurement only removes part of the singlet portion of the state, but the projective measurement removes both the S, T_0 component. Also, in the initial state of the cycle, there will be no T_0 component, so a different initial state is relevant. Repeating the steps in the minimal model section, starting with an initial state $(\beta, \gamma, 0, 0)$, taking the weak measurement on the singlet only but the projective measurement over both S and T_0, we find

$$P_{\text{tot}} = \sin^2\theta_1(1 - e^{-\Gamma t})|\beta + \gamma|^2/2 + |\beta + \gamma|^2(\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2 e^{-\Gamma t/2})^2/2 + |\beta - \gamma|^2 \sin^2(\theta_1 + \theta_2)/2. \quad (19)$$

Averaging this over a stochastic realization of the initial state preparation yields $\langle|\beta+\gamma|^2\rangle_r = \langle|\beta-\gamma|^2\rangle_r = 1/2$. Notice that the first two terms are the same as before, while the third term (arising from the T_0 projection) has no exponential suppression from the weak measurement.

B. Fast triplet-singlet admixing

The more interesting case is that of fast T_0, S admixing compared to the tunneling time, $\tau_{\text{admix}} \ll \Gamma^{-1} \ll T$. In this case, the T_0 and S components quickly oscillate into one another, and the tunneling process removes both the S and T_0 component in a symmetric way. This physics can be implemented by applying an exponential decay POVM, Eq. (5), to both S and T_0. Repeating the measurement dynamics analysis, it is straightforward to verify that the quantum system may be effectively represented by a two dimensional quantum system (qubit), rather than the four-level system above. This qubit represents the two possibilities of the single spins being parallel or antiparallel with one another (this can also be described as even or odd "parity").[10–13] Therefore, we can write an effective state $\psi = (\alpha, \beta)$, where α represents the parallel amplitude, while β represents the antiparallel amplitude.

The manipulation steps described above now read as follows: (i) The system always starts in the initial state $\psi = (1, 0)$, being in the spin-blockade regime [note that the initial state is the composite prepared state, not the "microscopic" (single-spin) one]. (ii) The ESR pulse on one spin then converts the spin-blockade state into the prepared initial state, $\psi_{\text{ESR}} = (\cos\theta_1, \sin\theta_1)$. (iii) The POVM elements in the (parallel-antiparallel basis) now take the simple form

$$M_0^\dagger M_0 = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Gamma t} \end{pmatrix}, \quad M_1^\dagger M_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 - e^{-\Gamma t} \end{pmatrix}, \quad (20)$$

implying

$$P_{iii}(1) = \sin^2\theta_1(1 - e^{-\Gamma t}),$$

$$P_{iii}(0) = \cos^2\theta_1 + \sin^2\theta_1 e^{-\Gamma t}. \quad (21)$$

If the tunneling occurs, the state is destroyed, while if the tunneling does not occur, the postmeasurement state is

$$\psi' = \frac{1}{\sqrt{\cos^2\theta_1 + \sin^2\theta_1 e^{-\Gamma t}}}\begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 e^{-\Gamma t/2} \end{pmatrix}. \quad (22)$$

Following Ref. 4 for this simple case, we introduce the angle ϑ, so the state (22) may also be written as

$$\psi' = \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix}. \quad (23)$$

(iv) The second ESR pulse (again on one spin only) is now applied with rotation angle θ_2, to produce the state

$$\psi_{\text{final}} = \begin{pmatrix} \cos(\vartheta + \theta_2) \\ \sin(\vartheta + \theta_2) \end{pmatrix}. \quad (24)$$

(v) Projecting on the antiparallel state, we find $P_v(1) = \sin^2(\vartheta + \theta_2)$, and $P_v(0) = \cos^2(\vartheta + \theta_2)$. This leaves the total probability of tunneling in the cycle as

$$P_{\text{tot}} = \sin^2\theta_1(1 - e^{-\Gamma t}) + (\cos^2\theta_1 + \sin^2\theta_1 e^{-\Gamma t})\sin^2(\vartheta + \theta_2). \quad (25)$$

In this expression, the preparation angle θ_1 and the tomography angle θ_2 provide a simple way of extracting the angle ϑ experimentally and verifying the theory of weak measurement in this system (as was similarly done in Ref. 4).

C. Quantum undemolition

In this same regime, $\tau_{\text{admix}} \ll \Gamma^{-1} \ll T$, it is also experimentally realistic to undo a measurement of an unknown initial state, "quantum undemolition" (QUD).[25] The idea follows that of the phase qubit introduced by Korotkov and one of the authors and relies on erasing the information obtained from the first measurement (for a popular version, see Ref.

26). The first two steps follow the prescription above: First, prepare the initial state with any angle θ_1, and make a weak measurement, characterized by the strength Γt. If no tunneling occurred, this brings us to the state (22). Next, swap the parallel and antiparallel amplitude with a π pulse on a *single spin*. Next, make a second weak measurement of the same strength, Γt, exactly as described above. Finally, a second π pulse swaps the amplitudes again back to the initial state. If the system did not tunnel in the first weak measurement, then the quantum state disturbance (22) occurred. If the system did not tunnel in the second weak measurement, then the quantum state disturbance of the first measurement (22) is undone, fully restoring the initial state $\psi_{\rm ESR}$ (even if this state is unknown). The probability for the QUD measurement to succeed, P_S, is simply the probability that the electron did not tunnel in the second measurement. Given that the state disturbance (22) did occur, the QUD success probability is

$$P_S = \exp(-\Gamma t)/(\cos^2\theta_1 + \sin^2\theta_1 e^{-\Gamma t}). \tag{26}$$

This means that a successful QUD measurement becomes less likely as the measurement strength increases.[27]

In order to confirm this theoretical prediction (for any initial state), it is necessary to make the further tomographic steps as in the minimal model section. This is carried out with a tomographic ESR pulse (that can be combined with the last π pulse) characterized by an angle θ_2, and a projective measurement on the antiparallel state.

The total probability of transporting one charge is the additive probability of tunneling in one of the three attempts described above, where each attempt probability is the multiplicative probability of tunneling at that time, but not at any previous step. Following a similar analysis as before, we find that the total probability of tunneling at any step is

$$P_{\rm tot} = 1 - e^{-\Gamma t} + e^{-\Gamma t}\sin^2(\tilde{\theta} + \theta_2). \tag{27}$$

The angle $\tilde{\theta}$ is again to be extracted experimentally (similar to ϑ previously). Here, we predict that $\tilde{\theta}=\theta_1$ if the measurement is undone.[25] The first term in Eq. (27) represents the possibility that tunneling occurs during the first weak measurement (no state disturbance to begin with), or the second weak measurement (a failed undoing attempt). The last term in Eq. (27) describes a successful QUD measurement, where the postmeasurement state of the undoing measurement coincides with the initial prepared state (regardless of our knowledge of it). Notice that the prefactor of the last term in Eq. (27), $e^{-\Gamma t}$, is interpreted as the QUD success probability (26) times $P_{iii}(0)$, the probability the initial state disturbance occurred in the first place (21).

Note that Eq. (27) recovers the correct limits: If $t=0$, the two π pulses simply undo each other, and the two angles add. As $t\to\infty$, there is always a transported charge: the first measurement removes the antiparallel component; the π pulse and the second measurement remove the parallel component.

Another interesting property of the undemolition sequence described above is that it can undo unitary errors that occur due to the presence of the nuclear spins, as described by Eq. (18) where the nuclear spins are treated classically and in the case when the external magnetic field exceeds the nuclear field. In that case, the nuclear field leads to a random phase φ between the parallel (upper) and antiparallel (lower) components of the state (22). The term $i\varphi$ is then simply added to Γt in the exponent in the antiparallel component, and is erased in much the same way as the Γt contribution due to the weak measurement.[28] This spin-echo-like effect has the added advantage that after many realizations, the other terms in Eq. (27) will be suppressed by averaging over the uncontrolled phase φ that will change from run to run, while the important undemolition term will remain, protected from the influence of the uncontrolled nuclear spins.

IV. DEPHASING AND SPIN RESONANCE ON BOTH SPINS

We now discuss in more detail the effect of dephasing on our results. We assume that the measurement time scales are much shorter than the ESR time scales, and therefore only include the effect of the Hamiltonian dynamics with the ESR manipulation (though the combination of both unitary and nonunitary dynamics is also very interesting, see Ref. 29). These unitary operations may be included into the analysis by operating with a generalized version of Eq. (13),

$$R_\alpha = \begin{pmatrix} \cos\theta_\alpha e^{i\phi_\alpha} & -\sin\theta_\alpha \\ \sin\theta_\alpha & \cos\theta_\alpha e^{-i\phi_\alpha} \end{pmatrix}, \tag{28}$$

where $\alpha=L,R$, the angle θ is the rotation angle about the y axis, and ϕ is the rotation angle about the z axis. In the two-spin (LR) Hilbert space, the above unitary operation on the left spin is given by

$$U_L = \begin{pmatrix} \cos\theta_L e^{i\phi_L} & 0 & 0 & -\sin\theta_L \\ 0 & \cos\theta_L e^{-i\phi_L} & \sin\theta_L & 0 \\ 0 & -\sin\theta_L & \cos\theta_L e^{i\phi_L} & 0 \\ \sin\theta_L & 0 & 0 & \cos\theta_L e^{-i\phi_L} \end{pmatrix}. \tag{29}$$

The same on the right spin is given by

$$U_R = \begin{pmatrix} \cos\theta_R e^{i\phi_R} & 0 & -\sin\theta_R & 0 \\ 0 & \cos\theta_R e^{-i\phi_R} & 0 & \sin\theta_R \\ \sin\theta_R & 0 & \cos\theta_R e^{-i\phi_R} & 0 \\ 0 & -\sin\theta_R & 0 & \cos\theta_R e^{i\phi_R} \end{pmatrix}. \tag{30}$$

The commuting matrices may be applied together with the ESR manipulations, and subsequently averaged over a Gaussian random distribution, whose width is controlled by the strength of the magnetic field fluctuations. A full analysis of dephasing is quite involved because statistically independent phases enter at every step in the procedure. Here, we present a simpler analysis that captures the basic physics. We consider the most important process of a dynamically changing z component of the nuclear magnetic field that affect both spins in the same way. The process is modeled by introducing a phase $\phi_L=\phi_R=\phi=(g\mu_B/2\hbar)\int_0^t B_N(t')dt'$ on both left and right spins, with $(\theta_1,\phi_1)_L$ and $(0,\phi_1)_R$ for the first

ESR pulse, and $(\theta_2,\phi_2)_L$ and $(0,\phi_2)_R$ for the second ESR pulse. We will then average over the phase $\phi(t)$ assuming uncorrelated white noise,[30] $\langle B_N(t')B_N(t'')\rangle=\sigma^2\delta(t'-t'')$, so that

$$\langle\phi(t)\phi(0)\rangle=(g\mu_B/2\hbar)^2\int_0^t dt'\,dt''\langle B_N(t')B_N(t'')\rangle$$
$$=(g\mu_B/2\hbar)^2\sigma^2 t=Dt, \quad (31)$$

where we introduced the dephasing rate, D.

Repeating the treatment in the minimal model section, starting with a general triplet state $(\beta,\gamma,\alpha,0)$ in the ST basis, we find that the total probability before any averaging is

$$\begin{aligned}P_{\mathrm{tot}}=&(1-e^{-\Gamma t})\sin^2\theta_1|\beta e^{i\phi_1}+\gamma e^{-i\phi_1}|^2/2\\&+|-\alpha\sin\theta_1\sin\theta_2(e^{i\phi_1+i\phi_2}-e^{-i\phi_1-i\phi_2})/\sqrt{2}\\&+\sin\theta_1\cos\theta_2 e^{-\Gamma t/2}(\gamma e^{-i\phi_1}+\beta e^{i\phi_1})\\&+\cos\theta_1\sin\theta_2(\beta e^{2i\phi_1+i\phi_2}+\gamma e^{-2i\phi_1-i\phi_2})|^2/2. \quad (32)\end{aligned}$$

If we now average over both initial state preparation, $\langle|\alpha|^2\rangle_r=\langle|\beta|^2\rangle_r=\langle|\gamma|^2\rangle_r=1/3$ (with vanishing averaged initial coherence), as well as the nuclear field, we end up with

$$\begin{aligned}\langle P_{\mathrm{tot}}\rangle=&[\sin^2\theta_1(1-e^{-\Gamma t})+\sin^2\theta_1\sin^2\theta_2 e^{-D\tau}\sinh(D\tau)\\&+\sin^2\theta_1\cos^2\theta_2 e^{-\Gamma t}+\cos^2\theta_1\sin^2\theta_2\\&+2\sin\theta_1\cos\theta_1\sin\theta_2\cos\theta_2 e^{-D\tau-\Gamma t/2}]/3, \quad (33)\end{aligned}$$

where $\tau=\tau_1+\tau_2$ is different from the weak-measurement time t. Here we see the presence of the T_0 term that coherently canceled before, as well as the suppression of the interference term that scales as $e^{-\Gamma t/2}$. Other types of dephasing will act similarly, suppressing all the terms in general.

V. CONCLUSIONS

We have developed a theory of weak quantum measurements for spin qubits. Inspired by a recent experiment demonstrating single-spin manipulation with ESR pulses in a double quantum-dot setup, we have shown how the current through such a device is affected by the fact that a quantum measurement of the spin state can be either weak or projective. The system is operated in the spin-blockade regime, where the spin singlet state contributes to transport and the three spin triplet states block it. A sequence of a state preparation step (using ESR), a weak-measurement step (using electron tunneling and spin-to-charge conversion), a state tomography step (using ESR), and a final strong projective measurement step (using electron tunneling and spin-to-charge conversion) is sufficient to exhibit a clear signature of quantum weak measurement in the current through the double quantum-dot system. We have analyzed how our results are affected by spin dephasing. As the major source of dephasing we have discussed the hyperfine interaction of the electron spin with the surrounding nuclear spins of the host semiconductor. This is a well established fact for GaAs quantum dots. We have shown that the combined effects of singlet-triplet mixing due to the nuclear field plus the consequences of weak-measurement theory yield interesting results in the regime where the singlet-triplet mixing is fast compared to the tunneling time of the weak-measurement step. In this regime, the weak measurement can even be undone and spin-echo technique is applicable in a straightforward way. We believe that our predictions can be readily observed in spin qubits formed, for instance, in GaAs quantum dots.

ACKNOWLEDGMENTS

We would like to thank Frank Koppens and Lieven Vandersypen for interesting and inspiring discussions. A.N.J. thanks Christoph Bruder and the Basel theory group for kind hospitality. This work was financially supported by the Swiss NSF and the NCCR Nanoscience.

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