Comment on "Three-Dimensional Anderson Localization in Variable Scale Disorder"

In a recent experiment [1], the expansion of noninteracting ultracold fermions was studied in a random speckle potential, and the observed density profiles were interpreted based on 3D Anderson localization. The purpose of this Comment is to demonstrate that slow diffusion of particles with a broad energy distribution and an energy-dependent diffusion coefficient leads to density profiles that agree with the measured data of [1], but not with the behavior expected for 3D Anderson localization.

Consider a nondegenerate Fermi gas prepared in a trap with the semiclassical position-energy distribution $F(\mathbf{r}, E)$. At time $t = 0$ the trap is switched off and atoms start to spread in the random potential. Assuming diffusive spreading, the ensemble averaged density is

$$\bar{n}(\mathbf{r}, t) = \int d\mathbf{R} \int dE P_E(\mathbf{r}, \mathbf{R}, t) F(\mathbf{R}, E), \qquad (1)$$

where the d-dimensional diffusion kernel $P_E(\mathbf{r}, \mathbf{R}, t) = (4\pi D_E t)^{-d/2} \exp(-|\mathbf{r} - \mathbf{R}|^2/4D_E t)$ propagates a particle with kinetic energy E and diffusion constant D_E from the initial position $\mathbf{R}$ to final position $\mathbf{r}$ in time t. In the long time limit, when the atoms have spread over distances much larger than the initial cloud, the starting point $\mathbf{R}$ in the kernel can be set to zero. The remaining integral over $\mathbf{R}$ yields the atomic energy distribution, $\int d\mathbf{R} F(\mathbf{R}, E) = C\nu(E) \exp(-\beta E)$, where C is a normalization constant, $\nu(E)$ is the density of states, and β measures the distribution width, generally due to temperature as well as disorder. Equation (1) then reduces to

$$\bar{n}(\mathbf{r}, t) = C \int dE \frac{\nu(E)}{(4\pi D_E t)^{d/2}} \exp\left(-\frac{r^2}{4D_E t} - \beta E\right). \qquad (2)$$

Thus, the density profile of a diffusing atomic cloud consists of a rather sharp central region, due to slowly diffusing particles piling up close to the center, followed by a (stretched)-exponential tail (see Ref. [2] for an analytically tractable example). Indeed, the tails of the profile (2) are determined by a saddle point of the expression in the exponent. When the energy dependence of the diffusion coefficient is a pure power law $D_E = \hbar E^a / m E_0^a$ (with E_0 an appropriate energy scale), this yields to leading order

$$\bar{n}(\mathbf{r}, t) \sim \exp\{-|r/s(t)|^{2/(a+1)}\}, \qquad (3)$$

valid for $r^2 \gg s(t)^2 = s_a^2 \hbar t / [m(\beta E_0)^a]$, with s_a of order unity. This is a stretched exponential with exponent $2/(a + 1)$ depending on dimensionality and type of disorder.

The experiment [1] uses a very strong and smooth random potential, with an axial correlation longer than the de Broglie wavelength of most atoms, such that $D_E \propto E^{5/2}$, over a sizable range of energies down to the rms potential strength [2]. The density tail is then the stretched exponential $|\log \bar{n}(\mathbf{r}, t)| \propto r^{4/7}$. It is important to note that Eq. (1) describes atoms in an intermediate energy interval. Atoms with higher energies appear ballistic in the finite field of observation, whereas atoms with lower energies should localize. The value $4/7$ for the exponent is just an estimate, although it lies in a reasonable range judging from the supplemental notes of [1]. Our main point here is that the density profiles of [1] are characteristic of a slowly diffusive component and are incompatible with the localization scenario which should lead to a power law tail (see below).

Even for the strongest disorder the data in Fig. 3(d) of [1] suggest that the rms radius of the cloud keeps growing instead of saturating on the observed time scale. Although it is difficult to determine an accurate slope, the residual dynamics could correspond to a diffusion coefficient of 40 μm^2/ms, which is much larger than the "quantum of diffusion" $\hbar/m \approx 1.5$ μm^2/ms of a potassium atom [2]. Thus, these data are compatible with diffusion at energies above the mobility edge. For some of the data in Fig. 3(d), occasionally, the size of the atomic cloud even appears to shrink. Systematic contraction is of course incompatible both with diffusion as well as localization, and should be analyzed with great care in view of the particle losses that limit the lifetime of the trapped gas—a task that is beyond the scope of the present Comment.

Last, there is an additional argument against a localization scenario. The cloud spreads to a rather large distance, of the order of 1 mm. If atoms are localized on such a large scale, they must originate from the critical energy interval next to the mobility edge, where the localization length diverges. But the resulting stationary density tail is well known to be a power law, $\bar{n} \sim r^{-(3+(1/\nu))}$ (here ν is the localization length exponent), instead of an exponential [2]. While this result was derived for a short-range, statistically isotropic potential, such power-law tails are expected to occur for any generic random potential, including anisotropic ones. Indeed, their existence is based solely on the fact that the localization length diverges as a power law when the mobility edge is approached from below, which is the generic behavior at the Anderson transition.

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