Probing entanglement via Rashba-induced shot noise oscillations*

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We have recently calculated shot noise for entangled and spin-polarized electrons in novel beam-splitter geometries with a local Rashba s-o interaction in the incoming leads. This interaction allows for a gate-controlled rotation of the incoming electron spins. Here we present an alternate simpler route to the shot noise calculation in the above work and focus on only electron pairs. Shot noise for these shows continuous bunching and antibunching behaviors. In addition, entangled and unentangled triplets yield distinctive shot noise oscillations. Besides allowing for a direct way to identify triplet and singlet states, these oscillations can be used to extract s-o coupling constants through noise measurements. Incoming leads with spin-orbit interband mixing give rise an additional modulation of the current noise. This extra rotation allows the design of a spin transistor with enhanced spin control.

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I. INTRODUCTION

Coherent control of entangled[1] and spin-polarized[2,3] electrons is relevant for the emerging fields of spintronics[4] and quantum computation.[4,5] Spin rotation of electron states is perhaps the simplest example of spin manipulation; and yet, it constitutes a relevant operation for quantum processing and the design of novel spintronic devices. Spin precession of electrically-injected carriers has recently been accomplished for diffusive transport in metallic wires;[6] however, this basic operation is still challenging for transport in ballistic channels.

The Rashba spin-orbit (s-o) interaction,[7] present in quantum-confined heterostructures lacking *structural* inversion symmetry, offers an interesting possibility to coherently rotate spin states in the *absence* of a magnetic field. This was first recognized by Datta and Das[8] in their spin-transistor proposal. Essential to this proposal is the electric control of the s-o interaction in ballistic one-dimensional channels.[9]

We have recently proposed[10] the use of a "local" s-o Rashba interaction as a means of modulating current and shot noise for spin-polarized and entangled electrons in novel beam-splitter geometries,[11,12] Fig. 1. The local Rashba interaction acting within an extension L of lead 1 allows for gate-controlled spin rotation of the incoming electrons.[13] This s-o induced spin rotation produces continuous changes in the symmetry of the spin part of the wave function which in turn affects the orbital motion and hence transport properties such as current and its fluctuations.

Here we present an alternate route to our shot noise calculation in a beam splitter with a local Rashba interaction.[10] We focus on triplet and singlet states only. We determine noise by first evolving the incoming electron states within lead 1 under the s-o interaction and then calculating the appropriate noise matrix elements involving these states. This less general approach allows us to determine shot noise in the presence of spin orbit straightforwardly in terms of the earlier results for bunching and antibunching in a beam-splitter geometry with no spin orbit.[14,15] We consider incoming leads with one and two modes; for the latter we also include a weak s-o induced interband coupling.[16] For entangled singlet and triplet states, we find *continuous* bunching and antibunching behavior as a function of the s-o–induced phase. In addition, our calculation shows that unentangled and entangled triplets display *distinctive* shot noise; this enhances the possibilities for entanglement detection

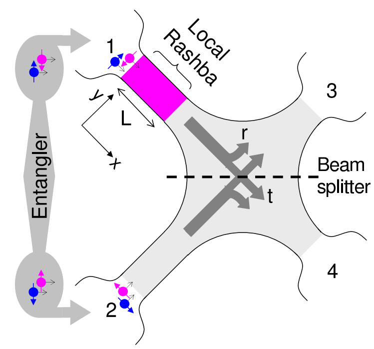

FIG. 1: Beam-splitter geometry with a local Rashba s-o interaction in lead 1. Entangled pairs are injected in leads 1 and 2. The portion of the entangled pairs traversing lead 1 undergoes a Rashba-induced spin rotation. This *continuously* changes the symmetry of the *spin* part of the pair wave function thus inducing sizable oscillations: continuous bunching and antibunching behaviors. (Adapted from Ref. [10].)

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schemes solely based on noise measurements. Spin-orbit interband mixing induces a further modulation on the noise for triplets and singlet. This is due to the coherent transfer of carriers between the s-o coupled Rashba bands as they traverse lead 1. This additional spin rotation allows the design of a spin transistor with enhanced spin control.[17]

This paper is organized as follows. In Sec. II we present our model Hamiltonian for one-dimensional channels with spin orbit. In Sec. III we define shot noise, outline our approach, and present results for beam-splitter configurations with and without s-o induced interband mixing in the incoming leads. In Sec. VI we briefly describe an extended Datta-Das transistor with additional spin control due to s-o induced interband mixing. Section V summarizes our conclusions.

II. SPIN-ORBIT COUPLING IN 1D CHANNELS

The beam-splitter leads in Fig. 1 are essentially ballistic quantum wires ("quantum point contacts"). Such one-dimensional channels can be defined by a gate-induced confining potential $V(y)$ on top of a two dimensional electron gas

$$H = -\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V(y), \tag{1}$$

m^* is the electron effective mass. The solution to the corresponding Schrödinger equation is

$$\varphi_{k_x,n,\sigma_z}(x,y) = \frac{e^{ik_x x}}{\sqrt{L_x}}\phi_n(y)|\sigma_z\rangle, \tag{2}$$

where $|\sigma_z\rangle$ denotes the electron spin component (σ_z basis), with eigenenergies

$$\varepsilon_{k_x,n,\sigma_z} = \frac{\hbar^2 k_x^2}{2m^*} + \epsilon_n,\ \ n = a, b... \ . \tag{3}$$

The transverse confining functions $\phi_n(y)$ satisfy

$$-\frac{\hbar^2}{2m^*}\frac{d^2\phi_n(y)}{dy^2} + V(y)\phi_n(y) = \epsilon_n\phi_n(y). \tag{4}$$

The confining potential can be chosen as either a parabolic or an infinite-barrier potential.

Rashba wire. Here we consider the Hamiltonian (1) with the additional Rashba term[7],

$$H_R = i\alpha\left(\sigma_y\partial_x - \sigma_x\partial_y\right). \tag{5}$$

where α is the s-o coupling constant and $\partial_i \equiv \partial/\partial i$, $i = x, y$. In a perturbative fashion, we can derive a reduced Hamiltonian for the problem by expanding the solution of the corresponding Schrödinger equation in the basis of the wire with no s-o $\left\{\varphi_{k_x,n,\sigma_z}(x,y)\right\}$. For two subbands we find[18]

$$H = \begin{bmatrix} \frac{\hbar^2k_x^2}{2m^*} + \epsilon_a & i\alpha k_x & 0 & -i\alpha d_{ab} \\ -i\alpha k_x & \frac{\hbar^2k_x^2}{2m^*} + \epsilon_a & -i\alpha d_{ab} & 0 \\ 0 & i\alpha d_{ab} & \frac{\hbar^2k_x^2}{2m^*} + \epsilon_b & i\alpha k_x \\ i\alpha d_{ab} & 0 & -i\alpha k_x & \frac{\hbar^2k_x^2}{2m^*} + \epsilon_b \end{bmatrix}, \tag{6}$$

where $d_{ab} \equiv \langle\phi_a|\partial/\partial y|\phi_b\rangle$ defines the 'interband coupling' in our system. For infinite-barrier confinement $d_{ab} = 8/3w$, w: is the width of the transverse channel. The Hamiltonian (6) can be easily diagonalized. Below we discuss the cases with and without interband coupling.

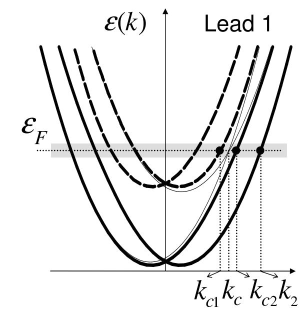

FIG. 2: Two Rashba bands with interband s-o mixing (solid and dashed thick lines). In absence of interband mixing (solid thin lines) the bands cross at particular k values (e.g., k_c). Spin-orbit induced interband coupling splits up the crossings. Impinging *spin up* electrons with energies ε_F near the band crossings (shaded region) are injected into Rashba states corresponding to the wave vectors k_{c1}, k_{c2}, and k_2.

A. Zero interband coupling ($d_{ab} = 0$): single spin rotation θ_R

Here the problem is strictly one dimensional; this should be a good approximation for αd_{ab} much smaller than the interband separation. In this case the solution is well known:[18] two uncoupled Rashba subbands since the Hamiltonian (6) is block diagonal for $d_{ab} = 0$

$$\varepsilon_{a,b}^{(s)}(k_x) = \frac{\hbar^2}{2m^*}\left(k_x - sk_R\right)^2 + \epsilon_{a,b} - \frac{\hbar^2k_R^2}{2m^*},\ s = \pm, \tag{7}$$

where $k_R \equiv m^*\alpha/\hbar^2$ is the Rashba wave vector. Note that in the absence of s-o induced interband coupling, the Rashba subbands cross, Fig. 2. The crossing for positive wave vectors occurs at

$$k_c = \frac{\epsilon_b - \epsilon_a}{2\alpha}, \tag{8}$$

which we obtain from $\varepsilon_a^-(k_c) = \varepsilon_b^+(k_c)$.

Spin-up electrons injected into either subband of the wire spin precess as they move down the channel. As first discussed by Datta and Das, a spin up electron traversing the length L of the Rashba region evolves to

$$|\uparrow\rangle \rightarrow \cos\theta_R/2|\uparrow\rangle + \sin\theta_R/2|\downarrow\rangle, \tag{9}$$

where $\theta_R = 2m^*\alpha L/\hbar^2$ is the precessing angle about the $-y$ axis. The above assumes a unity transmission[23] across the Rashba region.

We are particularly interested in incoming two-particle states, e.g., triplets and singlet pairs injected in leads 1 and 2, Fig. 1. The portion of these electron pairs crossing lead 1 undergoes a Rashba spin rotation. For instance, incoming singlet ($-$) and triplet ($+$) states yield

$$\begin{aligned}|\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle \rightarrow \quad & \cos\theta_R/2[|\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle] + \\ & \sin\theta_R/2[|\downarrow\downarrow\rangle \pm |\uparrow\uparrow\rangle].\end{aligned} \tag{10}$$

where we use the notation $|\uparrow\uparrow\rangle \equiv |\uparrow_1\uparrow_2\rangle$ to denote the electron spin state in the corresponding leads. Later on we will use the above states to determine the noise for injected spin entangled pairs. Equation (10) clearly shows a change in the symmetry of the spin part of the wave function of the pair: initially entangled triplet and singlet states at the entrance of the Rashba region emerge of it with s-o induced mixed in components of unentangled triplet states.

B. Non-zero interband coupling ($d_{ab} \neq 0$): additional spin rotation θ_d

In the presence of interband coupling the subband crossings in Fig. 2 split up, i.e., near the crossing point k_c the Rashba subbands anti cross. This is more easily seen by rewriting the Hamiltonian (6) in the basis of the $d_{ab} = 0$ problem. Instead of diagonalizing the problem exactly, here we follow a more intuitive perturbative approach similar to that of the nearly-free electron model.[17,20] For small interband couplings, we can approximate (lowest order) the energy dispersions near the crossing at k_c (Fig. 2) by

$$\varepsilon_\pm(k) = \frac{\hbar^2 k^2}{2m} + \frac{1}{2}\epsilon_a + \frac{1}{2}\epsilon_b \pm \alpha d, \tag{11}$$

with corresponding zeroth-order spinors ,

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}[|-\rangle_a \pm |+\rangle_b] = \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}_a \pm \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}_b, \tag{12}$$

where the subindices a and b denote the respective Rashba subbands.

How does an injected spin-up state with energy near the crossing gets rotated as it crosses the Rashba region? In analogy to the usual Datta-Das description for the case with no s-o interband coupling, we expand the incoming spin up state as

$$\begin{aligned}&|\uparrow\rangle_a e^{ikx}|_{x\rightarrow 0^-} = \\ &\quad \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{ik_{c1}x} + |\psi_-\rangle e^{ik_{c2}x}\right] + |+\rangle_a e^{ik_2 x}\right\}_{x\rightarrow 0^+}\end{aligned} \tag{13}$$

where $x = 0$ defines the entrance of the Rashba region.[23] Note that in the above expansion we only include *three* of the four intersection points in Fig. 2, $k_{c1} = k_c - \Delta/2$, $k_{c2} = k_c + \Delta/2$, with $\Delta = \theta_R d_{ab}/k_c$, and k_2 . The parameter Δ is obtained by imposing $\varepsilon_+(k_{c1}) = \varepsilon_-(k_{c2})$ near the crossing point k_c, i.e., for energies approximately equal to $\varepsilon_a^-(k_c) = \hbar^2 k_c^2/2m^* + \alpha k_c + \epsilon_a$. By substituting $|\psi_\pm\rangle$ [Eq. (12)] into Eq. (13) we can easily see that the boundary condition for the wave function is satisfied at $x = 0$. It is straightforward to verify that the boundary condition for the proper derivative of the wave function[19] is also satisfied, provided that $\Delta << 4k_F$; this condition is well satisfied for the parameters we use in our calculation. By following the same reasoning as for the uncoupled case Sec. (II-A), we find that an impinging spin-up electron in channel a evolves to the state

$$\begin{aligned}|\uparrow\rangle_a \;\rightarrow\; & \frac{1}{2}\left[\cos(\theta_d/2)e^{-i\theta_R/2} + e^{i\theta_R/2}\right]|\uparrow\rangle_a + \\ & \frac{1}{2}\left[-i\cos(\theta_d/2)e^{-i\theta_R/2} + ie^{i\theta_R/2}\right]|\downarrow\rangle_a - \\ & \frac{1}{2}i\sin(\theta_d/2)e^{-i\theta_R/2}|\uparrow\rangle_b + \\ & \frac{1}{2}\sin(\theta_d/2)e^{-i\theta_R/2}|\downarrow\rangle_b,\end{aligned} \tag{14}$$

upon crossing the Rashba region of length L in lead 1. Equation (14) clearly shows an additional modulation θ_d for electrons impinging near the band crossing, Fig. 2. Note that the above state vector yields the usual Datta-Das state (9) for $\theta_d = 0$. The extra spin-rotation $\theta_d = \theta_R d/k_c$ can, in principle, be varied independently of θ_R via proper lateral gating structures.

In the presence of s-o interband mixing, the portion of an electron pair crossing lead 1 undergoes a rotation θ_d in addition to the usual rotation θ_R. For instance, upon crossing the Rashba region in lead 1 an unentangled spin-up triplet emerges as

$$\begin{aligned}|\uparrow\uparrow\rangle_a \;\rightarrow\; & \frac{1}{2}\left[\cos(\theta_d/2)e^{-i\theta_R/2} + e^{i\theta_R/2}\right]|\uparrow\uparrow\rangle_{aa} + \\ & \frac{1}{2}\left[-i\cos(\theta_d/2)e^{-i\theta_R/2} + ie^{i\theta_R/2}\right]|\downarrow\uparrow\rangle_{aa} - \\ & \frac{1}{2}i\sin(\theta_d/2)e^{-i\theta_R/2}|\uparrow\uparrow\rangle_{ba} + \\ & \frac{1}{2}\sin(\theta_d/2)e^{-i\theta_R/2}|\downarrow\uparrow\rangle_{ba},\end{aligned} \tag{15}$$

where we use the notation $|\downarrow\uparrow\rangle_{ba} \equiv |\downarrow_{1b}\uparrow_{2a}\rangle$. Similar "evolved" pairwise states can be obtained for the singlet and the other triplets.

C. Realistic parameters

Before we proceed to determine shot noise, let us estimate the size of the spin rotations θ_R and θ_d. For the sake of simplicity and concreteness, let us assume an infinite (transverse) confining potential of width w. In this case, the s-o interband mixing matrix element is $d = 8/3w$ and $\epsilon_b = 3\pi^2\hbar^2/2mw^2$ ($\epsilon_a \equiv 0$, arbitrary origin). The Rashba energy $\epsilon_R \equiv \hbar^2 k_R^2/2m^*$ defines a natural scale in our system. Choosing $\epsilon_b = 16\epsilon_R$, we find $\alpha = (\sqrt{3}\pi/4)\hbar^2/mw = 3.45 \times 10^{-11}$ eVm (and $\epsilon_R \sim 0.39$ meV) for $m = 0.05m_0$ (see Ref. [9]) and $w = 60$ nm. In addition, the energy at the band crossing is $\varepsilon_a^-(k_c) = 24\varepsilon_R \sim 9.36$ meV; the Fermi energy should be tuned around this value. The wave vector at the crossing follows from Eq. (8), $k_c = 8\epsilon_R/\alpha$. A conservative estimate for the spin-rotation angles is obtained by assuming $L = 69$ nm; this yields $\theta_R = \pi$ and $\theta_d = \theta_R d/k_c = \pi/2$ since $d/k_c \sim 0.5$. In addition, we find $\Delta/4k_F \sim 0.05$ for the above parameters which assures that the bounday condition for the velocity operator is satisfied in our system [see Sec. (II-B)]. We emphasize the spin-rotation angles θ_R and θ_d can be independently varied. For instance, proper (side) gates can induce changes in the width w of the confining potential. Our simple estimates show that sizable spin rotations should be attainable experimentally.

III. SCATTERING APPROACH FOR CURRENT AND NOISE: BASICS

Definition. The discreteness of non-equilibrium charge flow gives rise to intrinsic fluctuations in the electric current: shot noise. Mathematically, shot noise is defined by the power spectral density of the current-current autocorrelation function. In a multi-lead geometry, the noise between leads γ and μ is

$$S_{\gamma\mu}(\omega) = \frac{1}{2}\int \langle \delta\hat{I}_\gamma(t)\delta\hat{I}_\mu(t') + \delta\hat{I}_\mu(t')\delta\hat{I}_\gamma(t)\rangle e^{i\omega t}dt, \tag{16}$$

where $\delta\hat{I}_\gamma(t)$ denotes the current fluctuation about its average in lead γ at time t. In the Landauer-Büttiker approach[21] the current in lead γ reads

$$\hat{I}_\gamma(t) = \frac{e}{h}\sum_{\alpha\beta}\int d\varepsilon d\varepsilon' e^{i(\varepsilon-\varepsilon')t/\hbar}\mathbf{a}^\dagger_\alpha(\varepsilon)\mathbf{A}_{\alpha,\beta}(\gamma;\varepsilon,\varepsilon')\mathbf{a}_\beta(\varepsilon'),$$
$$\mathbf{A}_{\alpha\beta}(\gamma;\varepsilon,\varepsilon') = \delta_{\gamma\alpha}\delta_{\gamma\beta}\mathbf{1} - \mathbf{s}^\dagger_{\gamma\alpha}(\varepsilon)\mathbf{s}_{\gamma\beta}(\varepsilon'), \tag{17}$$

where $\mathbf{s}$ denotes the scattering matrix of the system and $\mathbf{a}^\dagger_\alpha(\varepsilon)$ and $\mathbf{a}_\alpha(\varepsilon)$ are the usual fermionic creation and annihilation (two-component) operators for electrons; later on we write these more explicitly in terms of their spin components.

Beam splitter $\mathbf{s}$ *matrix.* The relevant scattering matrix in our problem is that of the beam splitter. We assume the beam splitter transmits electrons from leads 1 to 4 and from leads 2 to 3 with an amplitude t and from leads 1 to 3 and from 2 to 4 with an amplitude r. Hence

$$\mathbf{s} = \begin{pmatrix} 0 & 0 & \mathbf{s}_{13} & \mathbf{s}_{14} \\ 0 & 0 & \mathbf{s}_{23} & \mathbf{s}_{24} \\ \mathbf{s}_{31} & \mathbf{s}_{32} & 0 & 0 \\ \mathbf{s}_{41} & \mathbf{s}_{42} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & r & t \\ 0 & 0 & t & r \\ r & t & 0 & 0 \\ t & r & 0 & 0 \end{pmatrix}. \tag{18}$$

Note that backscattering into the incoming leads is neglected in $\mathbf{s}$. In addition, $\mathbf{s}$ is assumed to be both spin *and* channel independent.

In what follows we determine shot noise for electron pairs. In this case the angle brackets in the noise definition [Eq. (16)] should be interpreted as a quantum mechanical expectation value between entangled or unentangled electron states.

IV. SHOT NOISE IN THE ABSENCE OF SPIN ORBIT

A. Bunching and antibunching for entangled electrons: earlier results

Shot noise for singlet and triplets in a beam-splitter geometry with no spin-orbit interaction was first investigated in Ref. [14] within the scattering approach. These authors calculated the expectation value of the noise between singlet[24]

$$|S\rangle = \frac{1}{\sqrt{2}}\left[a^\dagger_{1\uparrow}(\varepsilon_1)a^\dagger_{2\downarrow}(\varepsilon_2) - a^\dagger_{1\downarrow}(\varepsilon_1)a^\dagger_{2\uparrow}(\varepsilon_2)\right]|0\rangle, \tag{19}$$

entangled triplet

$$|Te\rangle = \frac{1}{\sqrt{2}}\left[a^\dagger_{1\uparrow}(\varepsilon_1)a^\dagger_{2\downarrow}(\varepsilon_2) + a^\dagger_{1\downarrow}(\varepsilon_1)a^\dagger_{2\uparrow}(\varepsilon_2)\right]|0\rangle, \tag{20}$$

and unentangled triplets

$$|Tu_\sigma\rangle = a^\dagger_{1\sigma}(\varepsilon_1)a^\dagger_{2\sigma}(\varepsilon_2)|0\rangle, \ \sigma = \uparrow, \downarrow, \tag{21}$$

where $|0\rangle$ denotes the filled Fermi sea of the contacts and $a^\dagger_{\alpha\sigma}(\varepsilon_\alpha)$ [$a_{\alpha\sigma}(\varepsilon_\alpha)$] the creation (annihilation) operator for an incoming electron with energy ε_α in lead α; σ is the spin component along the quantization axis z. At zero temperature, zero frequency, and no applied voltage, the Fermi sea is completely noiseless and shot noise in the beam-splitter geometry is solely due to the injected pairs. The noise in lead 3 is found to be [14]

$$S_{33}^{S/Te,u_\sigma} = \frac{2e^2}{h\nu}T(1-T)(1 \pm \delta_{\varepsilon_1,\varepsilon_2}), \tag{22}$$

where $T = |t|^2$ denotes the transmission probability through the beam splitter and ν the density of states (discrete spectrum) in the leads. In Eq. (22) the upper sign refers to singlet and the lower one to triplet states. Note that all three triplets yield the *same* noise power.

The important result embodied in (22) is that singlet and triplet pairs give rise to shot noise bunching and antibunching, respectively. This terminology means that shot noise is enhanced for a singlet pair (bunching) while suppressed for triplets (antibunching) as compared to the shot noise for uncorrelated particles ("full shot noise"). The Fano factor in lead 3 is given by $F = S_{33}^{S/T}/2eI_3 = T(1-T)(1 \pm \delta_{\varepsilon_1,\varepsilon_2})$, where $I_\alpha = e/h\nu$ defines the average current in lead α. For uncorrelated particles the Fano factor is $F_{unc} = T(1-T)$. Note that the Fano factor for a singlet state is enhanced by a factor of two with respect to that of uncorrelated electrons while that of triplets is supressed to zero. As first pointed out in Ref. [14], this offers the possibility of detecting entanglement via shot noise measurements. As we discuss below, in the presence of a local Rashba interaction in lead 1 not only entangled states but also unentangled triplets can be distinguished via noise measurements.

V. SHOT NOISE IN THE PRESENCE OF SPIN ORBIT

The alternate route to the shot noise calculation we present below is simpler, though less general, than that of Ref. [10] in which an extended spin-dependent scattering formalism was used. Here we essentially evaluate the matrix element in the noise definition (16) by considering the appropriate Rashba-rotated states for one channel and two channels with s-o interband mixing.

A. One subband case: single modulation θ_R

For a single channel the relevant entangled states to the noise calculation are those in Eq. (10). Below we rewrite these in terms the singlet and triplet states in Eqs. (19)-(21)

$$|S/T_e\rangle_L = \cos(\theta_R/2)|S/Te\rangle + \frac{1}{\sqrt{2}}\sin(\theta_R/2)[|Tu_\downarrow\rangle \pm |Tu_\uparrow\rangle], \tag{23}$$

the above state clearly shows that the Rashba s-o interaction mixes up entangled and unentangled pairs as they cross the Rashba region of length L. Since the beam splitter $\mathbf{s}$ matrix is spin independent, the shot noise in lead 3 corresponding to $|S/T_e\rangle_L$ is simply

$$S_{s-o}^{S/Te_z}(\theta_R) = \cos^2(\theta_R/2)S_{33}^{S/Te} + \frac{1}{2}\sin^2(\theta_R/2)(S_{33}^{Tu_\downarrow} \pm S_{33}^{Tu_\uparrow}), \tag{24}$$

i.e., the noise in the presence of spin orbit can be expressed in terms of the earlier results with no spin orbit interaction, Sec. (IV-A). Substituting Eq. (22) in the above we find

$$\begin{aligned} S_{s-o}^{S/Te_z}(\theta_R) &= \frac{2e^2}{h\nu}T(1-T)[\cos^2(\theta_R/2)(1 \pm \delta_{\varepsilon_1,\varepsilon_2}) + \\ &\quad \sin^2(\theta_R/2)(1-\delta_{\varepsilon_1,\varepsilon_2})], \end{aligned} \tag{25}$$

which for the singlet simplifies to

$$S_{s-o}^{S}(\theta_R) = \frac{2e^2}{h\nu}T(1-T)[1+\cos(\theta_R)\delta_{\varepsilon_1,\varepsilon_2}], \tag{26}$$

and for the entangled triplet

$$S_{s-o}^{Te_z}(\theta_R) = \frac{2e^2}{h\nu}T(1-T)(1-\delta_{\varepsilon_1,\varepsilon_2}). \tag{27}$$

Note that the entangled singlet and triplet states are labelled with respect to the z axis. More symmetrical formulas can be obtained for a quantization axis along the Rashba rotation axis $(-y)$ (see Ref. [10]).

Similarly, the unentangled triplets along the z axis yield

$$S_{s-o}^{Tu_\uparrow}(\theta_R) = S_{s-o}^{Tu_\downarrow}(\theta_R) = \frac{2e^2}{h\nu}T(1-T)[1-\cos^2(\theta_R/2)\delta_{\varepsilon_1,\varepsilon_2}]. \tag{28}$$

Interestingly enough, the entangled and unentangled triplets along the z direction, Eqs. (27) and (28), display distinct shot noise as a function of the Rashba angle θ_R. This is in contrast to the case with no local Rashba interaction for which all triplets show identical noise [cf. Eq. (22), Sec. (IV-A)]. This feature makes it possible to distinguish unentangled from entangled triplet states via noise measurements as a function of the Rashba phase.

B. Two subbands + interband mixing: additional modulation θ_d

Here the portion of the injected pairs propagating in lead 1 undergoes both the ordinary Rashba rotation θ_R and the additional rotation θ_d due to interband coupling, Eq. (14). To determine shot noise in this case we proceed following the straightforward calculation in the preceding section; here, however, we use generalized spin-rotated states. For the spin-up unentangled triplet in Eq. (15), we find for the noise in lead 3

$$\begin{aligned} S_{s-o}^{Tu_\uparrow}(\theta_R,\theta_d) &= \frac{2e^2}{h\nu}T(1-T) \\ &\left[1-\frac{1}{2}\left(1-\frac{1}{2}\sin^2(\theta_d/2)+\cos(\theta_d/2)\cos\theta_R\right)\delta_{\varepsilon_1,\varepsilon_2}\right]. \end{aligned} \tag{29}$$

Note that $S_{s-o}^{Tu_\uparrow}(\theta_R,\theta_d) = S_{s-o}^{Tu_\downarrow}(\theta_R,\theta_d)$. Similarly, for entangled triplet and singlet states we find

$$S^{Te_z}(\theta_R,\theta_d) = \frac{2e^2}{h\nu}T(1-T)\left[1-\frac{1}{2}\left(\cos^2(\theta_d/2)+1\right)\delta_{\varepsilon_1,\varepsilon_2}\right], \tag{30}$$

and

$$S^{S}(\theta_R,\theta_d) = \frac{2e^2}{h\nu}T(1-T)\left[1+\left(\cos(\theta_d/2)\cos\theta_R\right)\delta_{\varepsilon_1,\varepsilon_2}\right], \tag{31}$$

respectively. Note the additional modulation θ_d due to s-o interband mixing in the above expressions. For $\theta_d = 0$ these reduce to the previous case [Eqs. (26) and (27)].

C. Some plots of the noise modulation: Fano factors

The average current in the incoming leads is $I_1 = I_2 = 2e/h\nu$. By normalizing the shot noise expressions (29), (30), and (31) by $2eIT(1-T)$ we obtain the "reduced" Fano factors

$$f_{Tu_\uparrow} = 1 - \frac{1}{2}\left(1 - \frac{1}{2}\sin^2(\theta_d/2) + \cos\frac{\theta_d}{2}\cos\theta_R\right)\delta_{\varepsilon_1\varepsilon_2}, \quad (32)$$

$$f_{Te_z} = 1 - \frac{1}{2}\left(\cos^2\frac{\theta_d}{2} + 1\right)\delta_{\varepsilon_1\varepsilon_2}, \quad (33)$$

$$f_S = 1 + \left(\cos\frac{\theta_d}{2}\cos\theta_R\right)\delta_{\varepsilon_1\varepsilon_2}. \quad (34)$$

Here, again, the triplets f_{Te_z} and $f_{Tu_\uparrow} = f_{Tu_\downarrow}$ show distinctive noise; the additional modulation θ_d makes them even more distinct [cf. Eqs. (33) and (34) to (26) and (27)]. Figure 3 illustrates the angular dependence of the Fano factor f_S for a singlet pair. Note the continuous bunching and antibunching as a function of the angles θ_R and θ_d; these angles are, in principle, independently tunable via a proper gating structure. The Fano factor for an entangled triplet pair along the Rashba rotation axis $(-y)$ displays a dependence similar to that in Fig. 3, i.e., $f_{Te_y} = 1 - \left(\cos\frac{\theta_d}{2}\cos\theta_R\right)\delta_{\varepsilon_1\varepsilon_2}$.

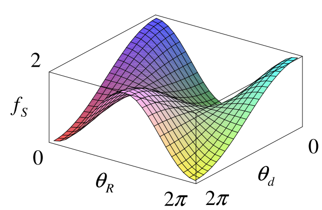

FIG. 3: Angular dependence of the Fano factor f_S for a single pair.

D. Extracting s-o coupling constants via noise measurements

The sizable Rashba-induced oscillations in the Fano factor plotted in Fig. 3 allows for a direct determination of s-o coupling constants via noise measurements since

$$\alpha = \frac{\hbar^2}{m^* L}\arccos\sqrt{f_S}, \quad (35)$$

from Eq. (32) for $\theta_d = 0$.

VI. A NEW "SPIN" ON DATTA-DAS TRANSISTOR?

Electrons impinging near the crossing. The additional spin rotation θ_d due to the transfer of electrons between s-o coupled bands allows for the design of a spin transistor with enhanced capabilities, i.e., with extra spin control. Let us consider a two-terminal geometry with spin-polarized emitter (source) and collector (drain) and a quasi-one-dimensional wire connecting them. We consider a wire with two-subbands in the presence of s-o induced interband coupling. As explained in Sec. (II-B), electrons transversing the wire undergo spin rotations θ_R and θ_d due to the Rashba interaction itself and the s–o induced interband coupling. Neglecting backscattering in the wire, we can easily determine the spin-resolved current from Eq. (14). Assuming a spin-up source, we find[17]

$$I_{\uparrow,\downarrow} \propto 1 \pm \cos(\theta_d/2)\cos(\theta_R). \quad (36)$$

For $\theta_d = 0$ Eq. (36) yields the usual current modulation in the Datta-Das transistor. We emphasize that the additional modulation θ_d occurs only for electrons with energies near the band crossing at k_c, see shaded region in Fig. 2. Electrons away from the crossing undergo the single rotation θ_R.

VII. CONCLUSION

A local Rashba spin-orbit interaction can strongly modulate both current and shot noise in novel beam-splitter geometries. For entangled electron pairs this interaction leads to tunable (continuous) bunching and antibunching behavior. Triplets states display distinctive noise as a function of the Rashba-induced phase; this allows one to distinguish triplets (entangled from unentangled) via noise measurements. These measurements can also extract spin-orbit coupling constants. Spin-orbit induced interband coupling in the incoming leads gives rise to additional spin rotation. This further modulates noise and current for electrons impinging near the band crossings. This feature should be relevant for the design of a spin-transistor with additional spin control. Our simple estimates indicate that sizable spin rotations are achievable for realistic system parameters.

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[23] Equation (13) assumes unity transmission through the Rashba region. The band-structure mismatch in our system is solely due to the Rashba energy $\epsilon_R \equiv \hbar^2 k_R^2/2m^*$ [see Fig. 1(b)] and is extremelly small since $\epsilon_R \ll \varepsilon_F$. We can estimate the transmission amplitude from $t = 2\,(1+\epsilon_R/\varepsilon_F)^{1/4}/[1+(1+\epsilon_R/\varepsilon_F)^{1/2}]$ (see Ref. [22]). For typical parameters[9] $\epsilon_R/\varepsilon_F = 0.1$ we find $|t|^2 = 0.99943$.

[24] The "delay time" τ_d between the injected partners in an electron pair is assumed negligible as compared to the "transit time" τ_t for an electron to cross the beam-splitter structure. For typical parameters we find $\tau_d \sim 0.6$ ps and $\tau_t \sim 10 - 100$ ps (see Ref. [10] for details).